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**REPORT**

## **A direct-reading chromaticity meter**

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### Summary

*The principles and outline design of a tristimulus spot colorimeter, which gives a continuously updated read-out of the chromaticity co-ordinates and luminance of a surface colour are described. The colorimeter can be used to measure automatically the colour of the light reflected from a flat surface or emitted by a television screen, and was primarily developed as a research tool for analytical investigations. A chromatic accuracy of 0.005 CIE-UCS units has been achieved together with an absolute luminance accuracy of  $\pm 5\%$ . Digital signal processing is used and the read-out is available in either of the standard (x, y) or (u, v) co-ordinates. In the principal mode of operation the readings are automatically updated every 1.2 seconds.*

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## 1. Introduction

The precise (objective) measurement of the colour of a self-luminous source or of an illuminated reflecting surface is a difficult procedure and can be time consuming. It is usually only worth undertaking for spectrally stable sources which can then be used and maintained as reference standards. It is quite easy, on the other hand, for the human eye to detect small differences in chromaticity, especially if the areas being compared subtend a large angular field, are close together, and have similar surface textures and luminances. No instrument exists which has a greater sensitivity than visual matching under optimum viewing conditions, or can provide measurements of the chromaticities being compared with sufficient accuracy to establish a fully reliable measure of their difference.

For analytical work associated with colour television broadcasting, however, the provision of a suitable visual reference for comparison is not always convenient and it is useful to have a portable colorimeter to hand. For example, one may wish rapidly to determine the correlated colour temperature of the lighting on a particular studio set, or the degree of saturation of a background paint, or perhaps the chromaticity of a monitor phosphor.

A portable Tri-Stimulus Colorimeter,<sup>1</sup> was developed prior to 1966 and several such instruments are still in use within the BBC and elsewhere. The earlier instrument, properly calibrated and used, can permit an accurate measurement of chromaticity, provided that the luminance of the colour remains sufficiently constant during the time interval of the measurement. The latter usually occupies between 10 and 20 seconds, because the instrument is manually operated and a total of six readings are required per measurement. Furthermore, in order to convert the instrument readings to an internationally accepted standard form of chromaticity co-ordinates, some arithmetic is necessary which, although not difficult, is slightly tedious and calculation errors can arise.

The work reported here relates to the subsequent development of a tri-stimulus spot colorimeter arranged to provide a continuously updated reading of the C.I.E. standard chromaticity co-ordinates (either  $x$ ,  $y$  or  $u$ ,  $v$ ) of the colour viewed. The main aim has been to establish a suitable method and then to assess the potential accuracy of the system by constructing a prototype instrument. No attempt was made to minimise the bulk and weight of the prototype instrument. Emphasis was placed on the achievement of high colorimetric accuracy and reliability and the ability of the instrument to measure the chromaticities of colours displayed on television cathode-ray-tube screens under normal operation; as discussed later, this type of scanned-phosphor source gives rise to additional photometric problems.

## 2. General principles

### 2.1 Colorimetry

The basis of additive colorimetry is that the colour of an object may be specified by measuring at least three independent quantities derived from the reflected (or emitted) light reaching the observer from the object. The quantities measured are spectrally-weighted integrations of the light intensity received by the colorimeter (analogous to the red, green and blue channel outputs in a three-tube colour television camera). The spectral-weighting functions can be arranged so that a linear transformation of the corresponding measured quantities give the  $X$ ,  $Y$ ,  $Z$  tristimulus values which relate to the 1931 C.I.E. reference primaries.<sup>2</sup> In this reference system, the  $Y$  value is directly proportional to the luminance (brightness) of the colour and the normalised quantities  $x = X/(X+Y+Z)$  and  $y = Y/(X+Y+Z)$  specify its chromaticity.

A recent (1961) set of primaries, based on a more uniform chromaticity scale (UCS), has been adopted internationally. In terms of these primaries, a colour is specified by the tristimulus values  $U$ ,  $V$ ,  $W$ . Here the  $V$  value is proportional to the luminance and the chromaticity co-ordinates are  $u = U/(U+V+W)$  and  $v = V/(U+V+W)$ . The principal difference with respect to the  $X$ ,  $Y$ ,  $Z$  system is that, when plotted on a chromaticity diagram, the vector lengths corresponding to just-noticeable-differences of chromaticity (at constant luminance) are more nearly uniform over the total colour gamut of real colours.

For either system, it is possible to generate the required set of tristimulus values directly, without recourse to an intermediate linear transformation process. However, this entails physically realising the overall spectral-weighting functions shown in Figs. 1(a) and 1(b), which refer to the ideal functions  $\bar{x}(\lambda)$ ,  $\bar{y}(\lambda)$ ,  $\bar{z}(\lambda)$  and  $\bar{u}(\lambda)$ ,  $\bar{v}(\lambda)$ ,  $\bar{w}(\lambda)$  respectively. It will be noticed that at least one of the functions in each system is bimodal; this type of spectral-weighting function is awkward to produce in practice by means of coloured glasses or similar filter materials. A typical, practical procedure for generating the bimodal  $\bar{x}(\lambda)$  function (see Fig. 1(a)) is to divide it into two unimodal subfunctions, to synthesize each part separately, and then subsequently to add the output values; thus a total of four measuring channels is involved. An alternative approximate method for the  $X$ ,  $Y$ ,  $Z$  system, which only requires the theoretical minimum of three independent measuring channels, is to use the longer wavelength unimodal part of the  $\bar{x}(\lambda)$  function and to add to fixed proportion of the  $Z$  value,  $kZ$  say. Because of the similarity in the shapes of the  $\bar{z}(\lambda)$  function and the short-wavelength part of the  $\bar{x}(\lambda)$  function, a value of  $k$  can be found which gives remarkably good agreement with the ideal  $X$  values over most of the chromaticity diagram.

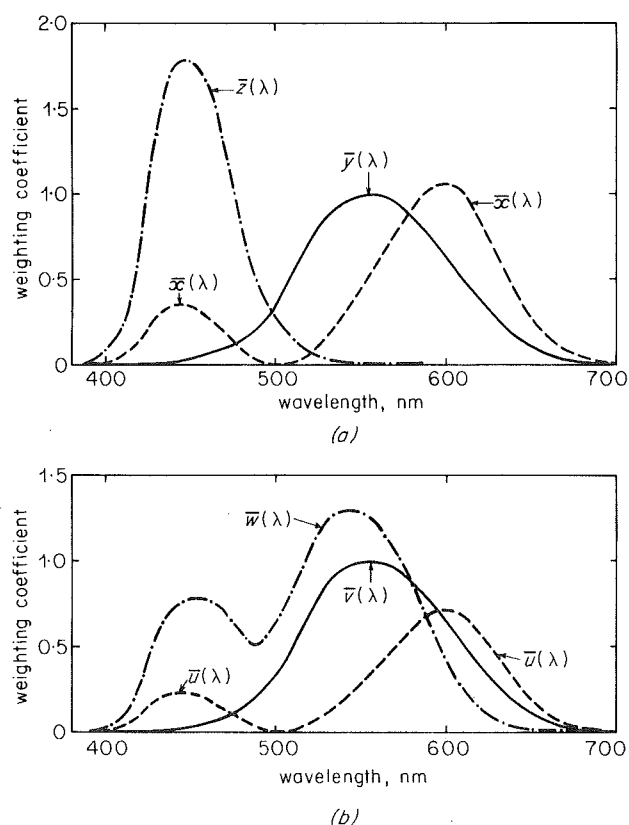


Fig. 1 - Ideal sets of spectral-weighting functions for the principal CIE reference systems  
(a) the (X), (Y), (Z) (1931) primary system  
(b) the (U), (V), (W) (1961) primary system

In the colorimeter described here, which continuously calculates, displays and updates the chromaticity readings, digital-logic processing of the colorimeter output signals was envisaged from the outset. The arithmetic processing proposals included a 3 X 3 linear matrixing operation to convert between the standard X, Y, Z and U, V, W systems. Advantage was taken of this matrixing facility to find an intermediate set of reference primaries which would result in spectral-weighting functions that were all substantially unimodal and (hopefully) were realisable with sufficient accuracy having ordinary colour-filter dyes. The intermediate set of primaries finally adopted, herein referred to as the (P), (Q), (R) primaries,\* are shown in Fig. 2(a) where their actual chromaticities are plotted on a (u, v) chromaticity diagram; the (X), (Y), (Z) and (U), (V), (W) sets of reference primaries are included for comparison together with the gamut of real colours bounded by the spectrum locus. The corresponding ideal spectral-weighting functions  $\bar{p}(\lambda)$ ,  $\bar{q}(\lambda)$  and  $\bar{r}(\lambda)$  are shown in Fig. 2(b).

It may be noted that all the systems discussed contain one common function which is, of course, the photopic

\*Note that the primary labelled (R) and its associated tristimulus value R should not be taken as referring to a red primary or red channel.

or visual sensitivity function.<sup>2</sup> Thus  $\bar{p}(\lambda) = \bar{v}(\lambda) = \bar{q}(\lambda)$  and either of the corresponding Y, V or Q output values could be used to determine the relative luminance of a colour.

Another potential advantage of a built-in matrixing facility is the possibility of making fine corrections for systematic discrepancies between the ideal spectral-weighting functions and the practical versions achieved in the colorimeter channels. The correction could be obtained by a slight adjustment of the pre-set matrix coefficients.

The basic relationships between the (X, Y, Z), (U, V, W) and (P, Q, R) sets of tristimulus values are given in the Appendix (1) to this report.

The final stage of the chromaticity calculation is a normalisation process in which the co-ordinates  $u = U/(U+V+W)$   $v = V/(U+V+W)$  etc. are formed. The tristimulus summations (U+V+W) and (X+Y+Z) required for these normalisations may be obtained directly by using the modified matrix relationships:

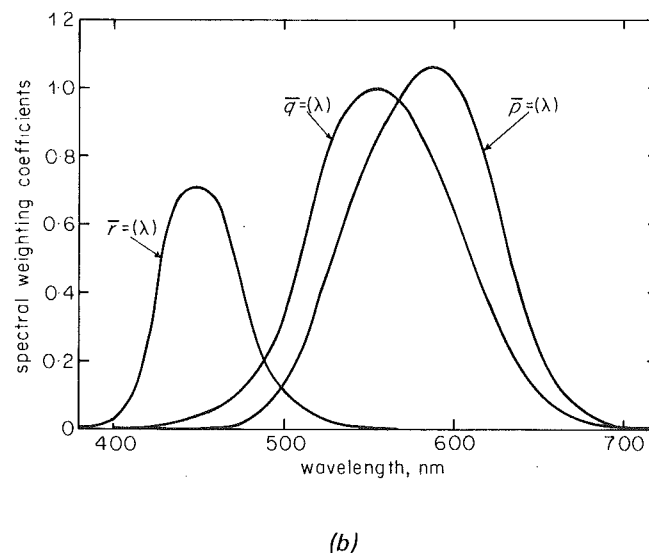
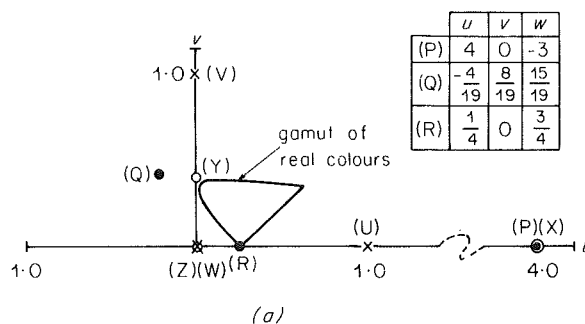


Fig. 2 - Chromatic properties of an intermediate set of reference primaries (P), (Q) and (R)

(a) chromaticities of primaries on a (u, v) chromaticity diagram  
(b) the corresponding ideal spectral-weighting functions  $\bar{p}(\lambda)$ ,  $\bar{q}(\lambda)$  and  $\bar{r}(\lambda)$



$$\begin{bmatrix} X \\ Y \\ (X+Y+Z) \end{bmatrix} = \begin{bmatrix} 1 & -\frac{5}{4} & \frac{1}{2} \\ 0 & 1 & 0 \\ -\frac{3}{2} & -\frac{11}{4} & 3 \end{bmatrix} \begin{bmatrix} P \\ Q \\ (P+Q+R) \end{bmatrix} \quad (1)$$

$$\begin{bmatrix} U \\ V \\ (U+V+W) \end{bmatrix} = \begin{bmatrix} \frac{2}{3} & -\frac{5}{6} & \frac{1}{3} \\ 0 & 1 & 0 \\ -\frac{13}{12} & \frac{25}{24} & \frac{4}{3} \end{bmatrix} \begin{bmatrix} P \\ Q \\ (P+Q+R) \end{bmatrix} \quad (2)$$

Suitably scaled versions of the above matrix coefficients are used in the prototype colorimeter. The  $(P+Q+R)$  value is obtained directly from the optical tristimulus generator described in Section 3.2

## 2.2 Mode of Operation

An ideal principle for a continuously-reading chromaticity meter is to use three optical channels and three photodetectors. The required tristimulus values of the values of the colour are then available simultaneously from the photodetector outputs. Consequently, the ratios of the tristimulus values, which determine the chromaticity of the colour, are not affected by any fluctuations of the luminance of the colour.\*

A serious practical drawback to this approach is the need to provide three separate photodetectors that have substantially identical characteristics with regard to speed of response, fatigue and drifts of gain with time and temperature. For an accurate instrument, continuously maintaining such a balance of characteristics would be difficult.

An alternative approach is to use three optical channels but only one photo-detector which is coupled to each channel sequentially, and this principle is used in the colorimeter described here. The separate signal-output values are stored until a complete set is available and the chromaticity calculation is then carried out. The rate at which the sequential signals are generated is an important factor. If it were too rapid, greater than 20 per second say, then strobing difficulties would arise when the instrument was used to measure surfaces illuminated by fluorescent discharge tubes or to measure television monitors. If the channel sampling rate were too slow, less than one every 5 seconds say, drifts and fluctuations in the luminous output of unstabilised sources would cause apparent chromaticity variations. Moreover, the system response would then be so sluggish that use of the instrument to adjust a colour to have a given luminance and/or chromaticity (e.g. setting the white point of a television monitor) would be tedious. Another factor is that, using alpha-numeric display tubes for visually reading out the calculated chromaticities, an updating period of between 1 and 2 seconds is regarded as about optimum.

\*Providing the chromaticity is not simultaneously fluctuating with the luminance.

A channel sampling-rate of approximately one every 0.3 seconds was chosen for the prototype colorimeter; a complete cycle of 3 channel samples plus one dark sample thus takes 1.2 seconds. This rate is low enough to be well clear of the luminance-fluctuation spectrum of a television display; consequently, the required signal values can be isolated by low-pass electrical filtering, providing there are no very-low frequency beat components (e.g. as might sometimes arise from asynchronous mains hum) present.

For dark (low-luminance) colours, the output signal-to-noise ratio tends to be limited by shot-noise in the detector. This is another reason for keeping the sampling rate and hence the system bandwidth, as low as possible.

After low-pass filtering, the signal from the detector is fed into an 10-bit analogue-to-digital (a. — d.) converter and the digital output is sampled, in accordance with the channel-selection arrangements, and stored (latched) until the next value for the same channel is obtained. The values so obtained represent the required tristimulus quantities and the digital calculation, which includes linear transformation of values as discussed earlier, occurs at the end of each complete channel-selection cycle (3 channel values plus 1 dark level).

## 3. Prototype design

### 3.1 Camera head

The purpose of the camera head is to provide a convenient means for selecting the particular area of surface to be measured. Light from the selected area must then be isolated and coupled to the colorimeter channels. Fig. 3 shows the arrangement adopted, which makes use of a conventional single-lens reflex camera mounted on a tripod stand. The camera body was modified, as shown, to accept a 5 mm diameter fibre-optic flexible light-guide. The entrance face of the guide coupling is mounted in the rear focal plane of the lens and determines the angular field of view for a given focal length of camera lens. The graph shown in Fig. 4 shows the angle of view and the equivalent field size at 2 metres from the lens, as a function of the focal length of the camera lens: an angle of 2° is a very convenient size for laboratory work and this is obtained with a 135 mm telephoto camera lens.

The purpose of the light-guide is two-fold. First it allows the relatively bulky tristimulus generator unit (described in Section 3.2) to be separated from the camera head assembly. (For example, the tristimulus generator unit could be located at the base of a tripod). Thus, positioning the camera head in order to select the desired area for measurement is an easy operation and the final position can be firmly locked. Secondly, in principle, advantage can be taken of the spatial incoherence of the fibre locations at the two ends of the light-guide to obtain good integration of light flux received from the selected area for measurement. This helps to avoid systematic errors which could arise from say, an unevenness in the luminance over the selected area.

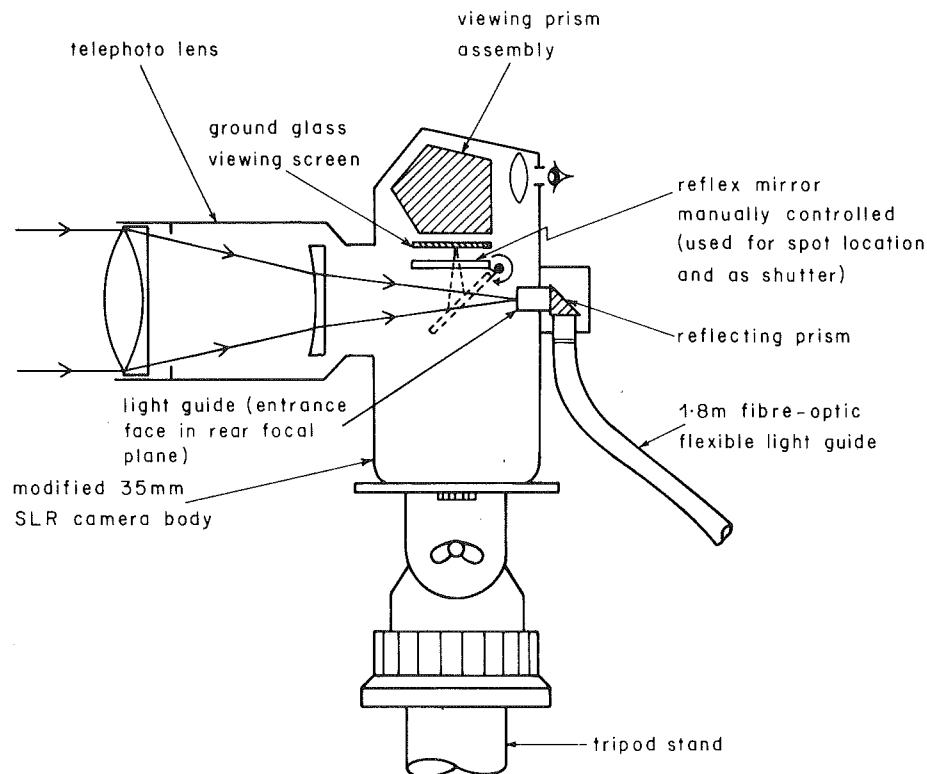


Fig. 3 - Camera — head arrangement

The light-guide\* used for the prototype was 1.8 metres long and 5mm diameter. The limiting f-number of the individual fibres was approximately  $f/1.2$ . It was found, however, that a practical operating value for the complete camera-head assembly was  $f/2.8$ . Any further optical attenuation, if necessary for very bright sources, is best carried out either by means of aperture-grid filters or by inserting neutral metal-filters, rather than by reducing the camera aperture.\*\*

### 3.2 Tristimulus generator

As the name implies, the main purpose of this unit is to generate sequences of electrical signals which are proportional to the tristimulus values of the colour being measured. It comprises a 3-way light-splitting and colour filtering assembly, a mechanical scanning arrangement and a photomultiplier detector with its attendant E.H.T. supply. The unit also houses electrical filtering, opto-electronic synchronisation, amplification and gain-control arrangements. These interface assemblies are described in detail in Section 3.3.

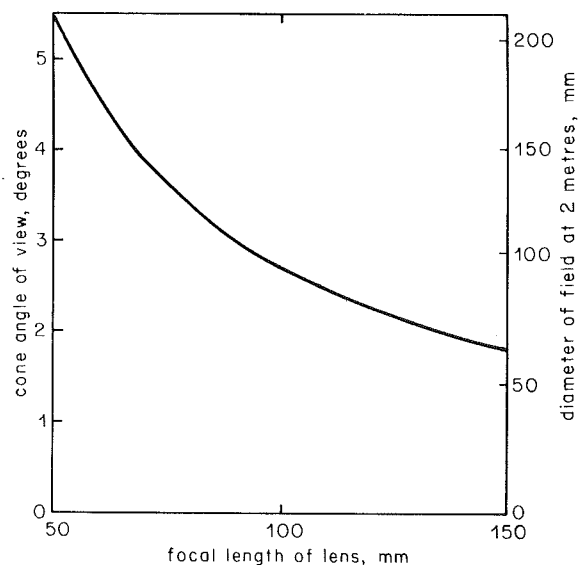


Fig. 4 - Field of view of the camera head as a function of the focal length of the lens: left-hand ordinate scale gives the angular field in degrees, right-hand ordinate scale gives the diameter of the circular field at 2 metres from the camera

\*Standard product of Barr and Stroud Ltd.

\*\*This is believed due to unwanted spatial-coherence effects found in the particular light-guide used.

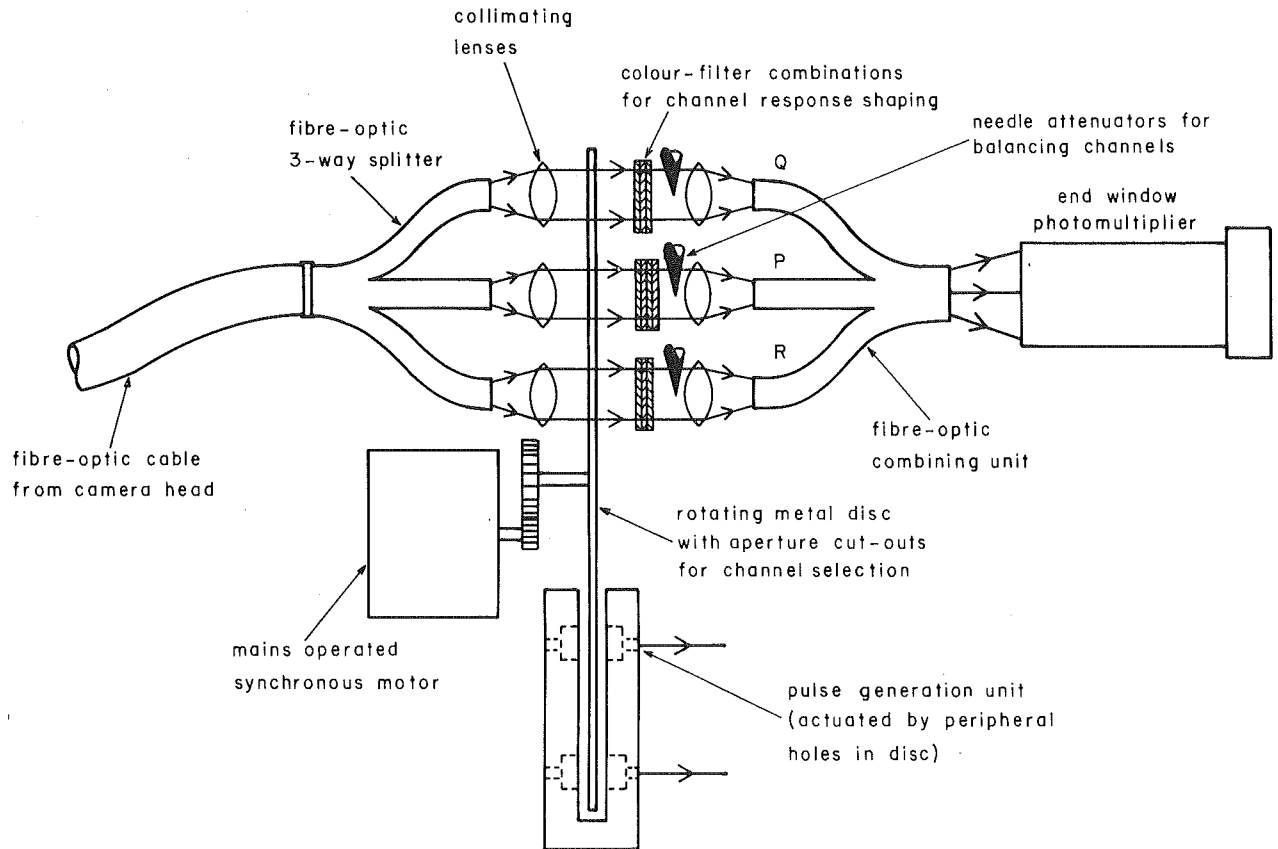


Fig. 5 - Tristimulus generator arrangement

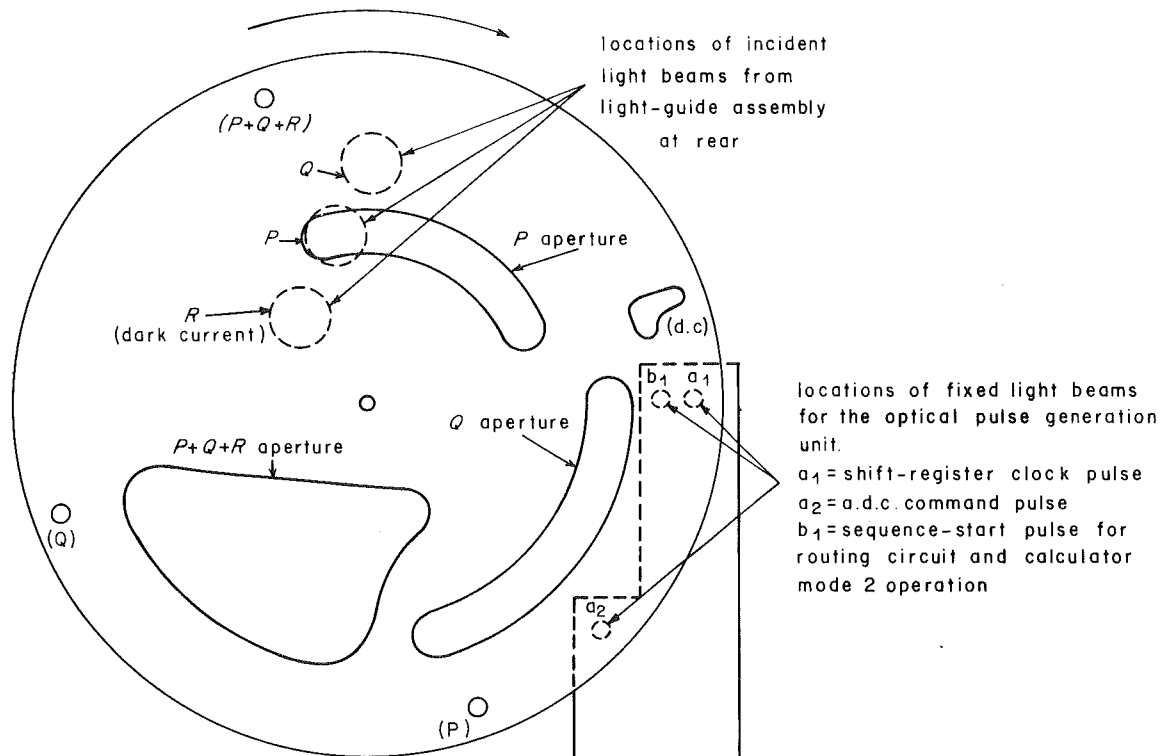


Fig. 6 - Aperture arrangement in scanning disc

### 3.2.1 Optical/Mechanical arrangement

The light emerging from the flexible fibre-optic guide is coupled to a rigid fibre-optic component, housed within the unit, which divides the flux equally (approximately) between three channels. The general arrangement of the component parts is illustrated in Fig. 5. Each of the optical channels must be spectrally weighted, as described below, and this is achieved by inserting absorption-type glass or glass/gelatine colour-filter combinations. Lenses in each optical channel are used for collimating the light prior to passage through the colour filters and for approximately re-focussing it onto the end faces of a second similar fibre-optic component used for re-combining the beams. Tapered needle attenuators, externally adjusted by turning a screw, are inserted in each optical channel for initially balancing the channels. A rotating metal disc interrupts each of the three optical channels, in accordance with the pattern of disc apertures shown in Fig. 6. The disc is driven through step-down gearing from a small synchronous motor (50 Hz, 240V) and rotates once every 1.2 seconds.

It will be seen (Figs. 5 and 6) that the disc apertures control the proportion of the cycle time that a given channel or group of channels is coupled to the photo-detector. The coupling sequence used was channel *P*, channel *Q*, channels (*P* + *Q* + *R*), no channel (i.e. dark), with approximately equal-exposure periods of 0.3 seconds.\*

The disc also contains small holes around its periphery which in combination with solid-state lamps (L.E.D.'s) and photoswitches, produce timing control pulses rigidly synchronised to each channel sampling period and to the complete cycle period.

The exit face of the fibre-optic combining unit is coupled into the housing containing a 30 mm diameter photomultiplier tube with an end-window photocathode; the tube is magnetically shielded by a concentric Mu-metal cylinder. After consideration of the various types of photomultiplier tube available, one with an S.10 photocathode response and eleven stages of electron multiplication was selected and is operated at an overall dynode voltage between 600 and 1000 volts, using a small EHT supply stabilised by a chain of 150V Zenner diodes. The gain of the tube is a sensitive function of the inter-dynode voltages and, to accommodate a large range of input light intensities, these voltages can be changed.

### 3.2.2 Colour-filter design

The basic colorimetric accuracy of the instrument largely determined by the degree to which the actual spectral responses of the three channels approximate those of the ideal characteristics. In the present case, an attempt

\*The output signal corresponding to the sum of the three channels (*P* + *Q* + *R*) is clearly the largest of the sequence and it was originally intended to use this 'peak' signal as a reference for an automatic digital gain control. The latter concept was not implemented, however, in the prototype.

was made physically to realise the ideal  $\bar{p}(\lambda)$ ,  $\bar{q}(\lambda)$  and  $\bar{r}(\lambda)$  functions given in Section 2.1 (Fig. 2).

The spectral-weighting  $S(\lambda)$  introduced by the various optical components used in the colorimeter (camera lens, fibre-optic guide, collimating lenses etc), including the photomultiplier, must be measured with some care before the shaping filter characteristic  $F(\lambda)$  required in each channel can be determined. Ideally, at each wavelength  $\lambda$ ,

$$\begin{aligned} C_p F_p(\lambda) &= \bar{p}(\lambda) / S(\lambda) \\ C_q F_q(\lambda) &= \bar{q}(\lambda) / S(\lambda) \\ C_r F_r(\lambda) &= \bar{r}(\lambda) / S(\lambda) \end{aligned} \quad (3)$$

where  $C_p$ ,  $C_q$  and  $C_r$  are area normalising constants independent of wavelength, and the subscripts *p*, *q* and *r* denote the three channels respectively.

In each case the required shaping filter characteristic is provided by a suitable combination of gelatin-type and/or glass-type colour filter materials. These types of filter rely on the selective absorption of transmitted light by chemical dyes uniformly dispersed within the media. They are known to obey Bouguer's Law, i.e., that the internal optical density of the filter is directly proportional to its thickness. Glass colour filters are preferred because a sensitive control of spectral transmission characteristic can be obtained by varying the filter thickness. Moreover, the permanence and optical quality of glass-type materials is generally superior to gelatin or acetate based materials.

There are, unfortunately, only a few basic dye elements that can be used in practical glass filters. Thus spectral matching is never ideal, but very close approximations to some functions can be obtained by a suitable choice of colour filters and photodetector response.<sup>3</sup> In practice, therefore, there is an error function  $E(\lambda)$  associated with each channel response. For example, in the *p* channel we obtain  $\bar{p}(\lambda) + E_p(\lambda)$  instead of the ideal  $\bar{p}(\lambda)$  function; this represents a fractional error of  $E_p(\lambda)/\bar{p}(\lambda)$ . The basic problem here is to optimize the spectral characteristic of the shaping filter  $F(\lambda)$  in such a way that the final error function  $E(\lambda)$  has the least effect on the overall instrumental accuracy. There is, of course, no unique solution to this problem, and optimisation procedures are usually limited to a particular set of test colours (e.g. the monochromatic spectrum colours).

However, various selection and optimisation techniques using a computer were devised to decide the best types and thicknesses of the glass filters.\*

The starting point was to compile a data bank for the spectral density characteristics of a reasonable number of commercially-available colour glasses and gelatin or acetate filters. For glass types, the spectral density information at a nominal thickness of 1 mm was stored. The next stage was to determine the filter function required (see equation 3) and select those glass or gelatin

\*R.W. Lee was responsible for writing the computer programme

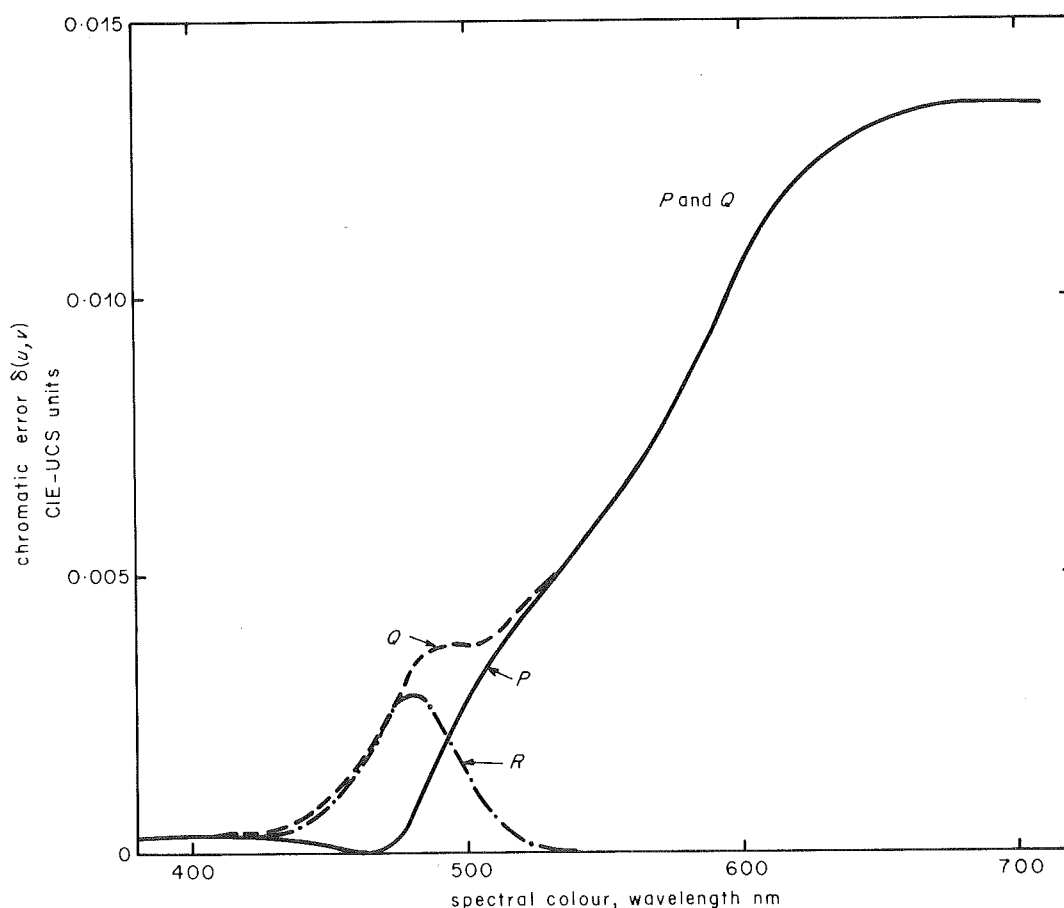


Fig. 7 - Calculated chromaticity errors ( $u, v$ ) for narrow-band spectrum colours assuming a 2% error in the response of any one of the three channels (as indicated)

———— error in P channel,      - - - - - error in Q channel,      - . . . - error in R channel

filters listed in the bank which would be potentially useful. This was carried out by the computer on the basis of least-square density errors averaged over the wavelength range of interest for a given channel. For small density errors (less than 0.05 say) this criteria is equivalent to selecting the minimum average value of  $[E(\lambda)/\bar{p}(\lambda)]^2$ . The discrete thicknesses of the glass filters considered were 0.5, 1.0, 2.0, 3.0, 4.0mm for the purpose of the initial selection process.

Having selected a number of potentially useful filters for various wavelength ranges, the next stage consisted of trying combinations of these filters. This step is necessary because, for broad-band matching, it is unlikely that a single filter can provide a sufficiently good fit over the whole wavelength range. The result of this computer processing is a tabulation of the filter combinations, their coarse optimum thickness (in 0.5 mm steps for glass, or number of pieces for gelatin) and their best-fit measure in terms of root-mean-square of density errors. The computer also provides the value of the constant,  $\log(1/C)$ , where  $C$  is as defined in equation (3). This constant is a measure of the mean density of a channel and thus indicates the

relative photometric efficiency of the channel. A high value of  $C$  may, in fact, rule out many of the combinations listed simply on efficiency grounds in spite of a favourable colorimetric performance.

At the combinations stage of the optimisation programme, there is provision for introducing weighting-factors for the density errors as a function of wavelength. It has already been mentioned that a small density error at a given wavelength is proportional to the fractional error in the final chromaticity; it depends in fact on the spectral characteristics of the particular colour. This is shown rather forcibly in Fig. 7, for example, where the resulting chromaticity errors of the (monochromatic) spectrum colours are plotted, assuming a 2% error in the response in any one of the three channels (as indicated). The general colorimetric equations relating to these calculations are given in Appendix (2). It is clear from Fig. 7 that the effect of a response error on colours in the red part of the spectrum is far more significant than that of an equivalent error for colours in the cyan and blue parts of the spectrum. However, these results refer to an extreme set of narrow-band test colours, i.e. the spectrum

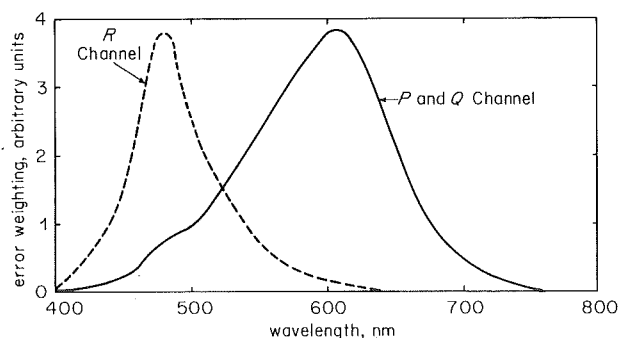


Fig. 8 - Error-weighting functions used in the filter optimisation program

—— P and Q channels      ---- R channel

colours, and the situation is somewhat different for broadband desaturated colours. In the latter case, the fractional error at a given wavelength tends to affect the final chromaticity in proportion to the actual value of  $\bar{p}(\lambda)$ ; thus much larger fractional errors can be tolerated in the tails of the channel responses than near their peaks. The error-weighting functions used in the computer optimisation programme are an arbitrary combination of the two cases discussed. They are shown in Fig. 8., where it will be seen that, for the P and Q channels, the density errors in the wavelength region  $\lambda = 500$  nm to  $\lambda = 680$  nm are given the greatest weight; on the other hand, for the R channel, emphasis is placed on errors in the range  $\lambda = 440$  nm to  $\lambda = 540$  nm.

The final stage in the filter design is to optimise the thicknesses of the glasses in the chosen combination in order to achieve the best (weighted) fit. Because the intrinsic transmission characteristics of glasses, of the same nominal type, vary in production slightly from melt to melt, it was found necessary to obtain the required glass blanks and carefully measure their characteristics. This data was then used in the final optimisation programme, and the blanks were then ground to the computed thicknesses and finally polished.

The colour filters used for each channel were cemented together into a single unit in order to eliminate interfacial reflection losses.

### 3.3 Interface arrangements

#### 3.3.1 Amplification and filtering

As already mentioned (section 3.2.1) the light detector is a photomultiplier tube. The particular tube used was carefully selected for low dark-current and adequate gain stability; it was also found necessary to work with collector output currents less than two micro-amperes in order to reduce long-term fatigue effects.

Fig. 9 shows the interface circuit used to match the photomultiplier signal output to the analogue-to-digital converter. The collector (anode) current is fed to two differential integrated-circuit amplifiers in series via a low-pass filter. The latter, together with capacitive elements in the amplifier feedback circuits, serves to attenuate frequency components above 3 Hz.

The basic gain of the detector is varied in six discrete steps by switching in different series resistors in the dynode-chain circuit. This varies the overall dynode voltage from 500 volts to 1200 volts, producing a corresponding gain change of X1 to X300. The load resistors are adjusted so that, in conjunction with a decimal-point shift in the numerical read-out and a 'divide by three' indicator lamp, the absolute luminance can be correctly ascertained for any position of the range-change switch. A small change in the overall gain of the amplifiers, for a final calibration of the luminance reading, is possible by means of a variable (pre-set) resistor in the feedback circuit of the second differential amplifier.

The analogue-to-digital convertor requires a 5 volt signal input for maximum (i.e. 10 bit) resolution. A larger signal would overload the convertor and a false chromaticity reading would then be displayed. To indicate this overload condition, a comparator circuit has been included which switches on a lamp on the control panel whenever the input voltage to the a.d.c. exceeds 5 V.

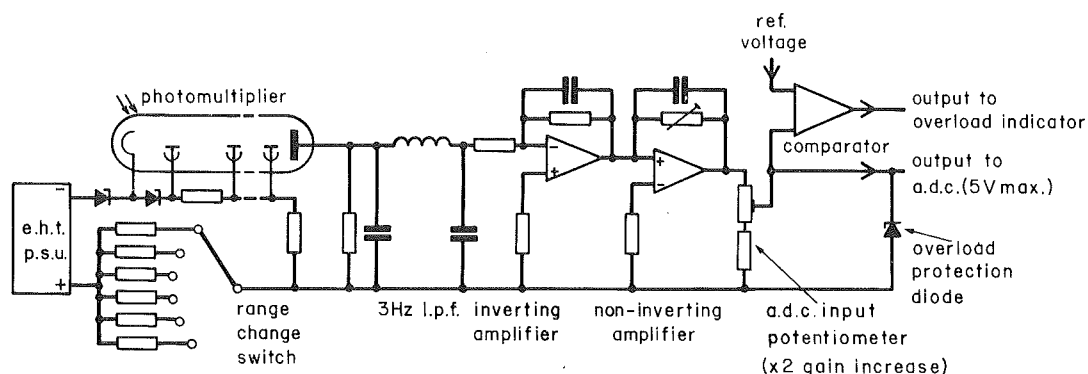


Fig. 9 - Interface circuitry

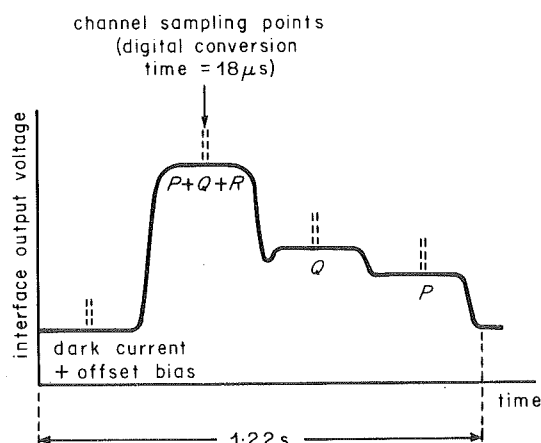


Fig. 10 - Typical analogue voltage cycle at the input to the a.-d. convertor

A peak signal-level which happens to occur in the region of 2 volts, i.e. at the low end of a given range, will not be fully resolved and this may result in chromaticity readings which are not within the 0.005 CIE-UCS unit specification. This situation can be overcome when necessary by means of the a.-d. convertor input potentiometer (see Fig. 9) which is mounted on the main control panel. A clockwise rotation of the potentiometer increases the input signal level to the a.-d. convertor by up to a factor of two. It should be noted that if the input potentiometer is used (this is indicated by a panel lamp) the luminance reading is then uncalibrated.

### 3.3.2 Analogue-to-digital conversion and pulse generation

Fig. 10 shows a typical analogue waveform for one rotation of the scanning disc at the input to the a.-d. convertor; consecutive voltage levels correspond to 'dark current',  $P+Q+R$ ,  $Q$  and  $P$  values. Simultaneously, timing pulses for sampling and clock synchronising are generated by the opto-electronic switches positioned near the periphery of the scanning disc and actuated by suitably positioned perforations (see Fig. 6).

Fig. 11 gives the circuit arrangement used for routing and storing the digital signals after conversion. The ten parallel outputs from the a.-d. convertor are routed in sequence to four sets of bistable latches which store the converted signal values, normally for one period of the scanning disc. Clock pulses for actuating these latches are obtained from the 'status' output of the a.-d. convertor to ensure that complete conversion has taken place before storing the information in the latches. The a.-d. convertor is itself controlled by 'convert' command pulses originated by the scanning disc. A command pulse (one per channel) is so timed that conversion, which takes only  $18 \mu\text{s}$ , is carried out when the corresponding optical channel is completely unobstructed. This sampling process is illustrated in Fig. 10.

The outputs of four logic gates are used to actuate the bistable latches in sequence and the latching sequence is generated by the output of a shift register which, in turn, is controlled by a sequence-start pulse followed by subsequent clock-pulses, all initiated by the scanning-disc pulse generating unit. The shift register sequence operation is monitored by visual indicators on the main display panel. The sequence-start pulse is also used for initiating the logic calculator, in its principal operating mode.

## 3.4 Logic processor

### 3.4.1 Chromaticity calculation

A block diagram, which indicates the basic steps in the calculation of the chromaticity co-ordinates, is shown in Fig. 12. The T.T.L. range of logic elements is used throughout.

The first step is a simple subtraction process, the purpose of which is to correct the principal signal values ( $P+Q+R$ ,  $Q$ ,  $P$ ) by deducting any bias resulting from the photomultiplier dark current as well as any positive offset voltage from the operational amplifiers.

The next step is to carry out a  $3 \times 3$  linear transformation of the input values to obtain either the CIE (1931) standard set of tristimulus quantities,  $X$ ,  $Y$ ,  $X+Y+Z$  or the CIE (1961) 'uniform chromaticity' set of quantities  $U$ ,  $V$ ,  $U+V+W$ . The two sets of matrix coefficients necessary to execute the transformation are held in pre-set stores (in 5-bit form) within the calculator; a single switch on the control panel serves to present either set of coefficients as constant multipliers. Each bit of the current tristimulus value is serially multiplied by the appropriate pre-set coefficient and individual products formed. This process is repeated for all the coefficients and a final summation carried out, taking account of the 'sign' of the coefficient, to obtain each of the transformed values.

Having transformed the tristimulus values, the divisions  $X/(X+Y+Z)$  and  $Y/(X+Y+Z)$  or  $U/(U+V+W)$  and  $V/(U+V+W)$  are carried out to form the chromaticity co-ordinates  $x$  and  $y$  or  $u$  and  $v$ , respectively. The division process comprises a straight-forward series of shift-or-subtract operations, depending on the relative sizes of the denominator and (current) numerator.

The resulting chromaticity co-ordinate values, being less than unity, cannot be directly decoded to decimal form for subsequent display. They are therefore converted to integer form by multiplying (in binary) by one thousand, and the resulting display thus gives each co-ordinate to 3 decimal places.

### 3.4.2 Decoding and display

The binary values (now scaled by  $10^3$ ) are next converted into binary-coded decimal (B.C.D.) form for display using seven-segment alpha-numeric indicators. The indicators are driven by B.C.D.-to-seven-segment decoder-drivers.

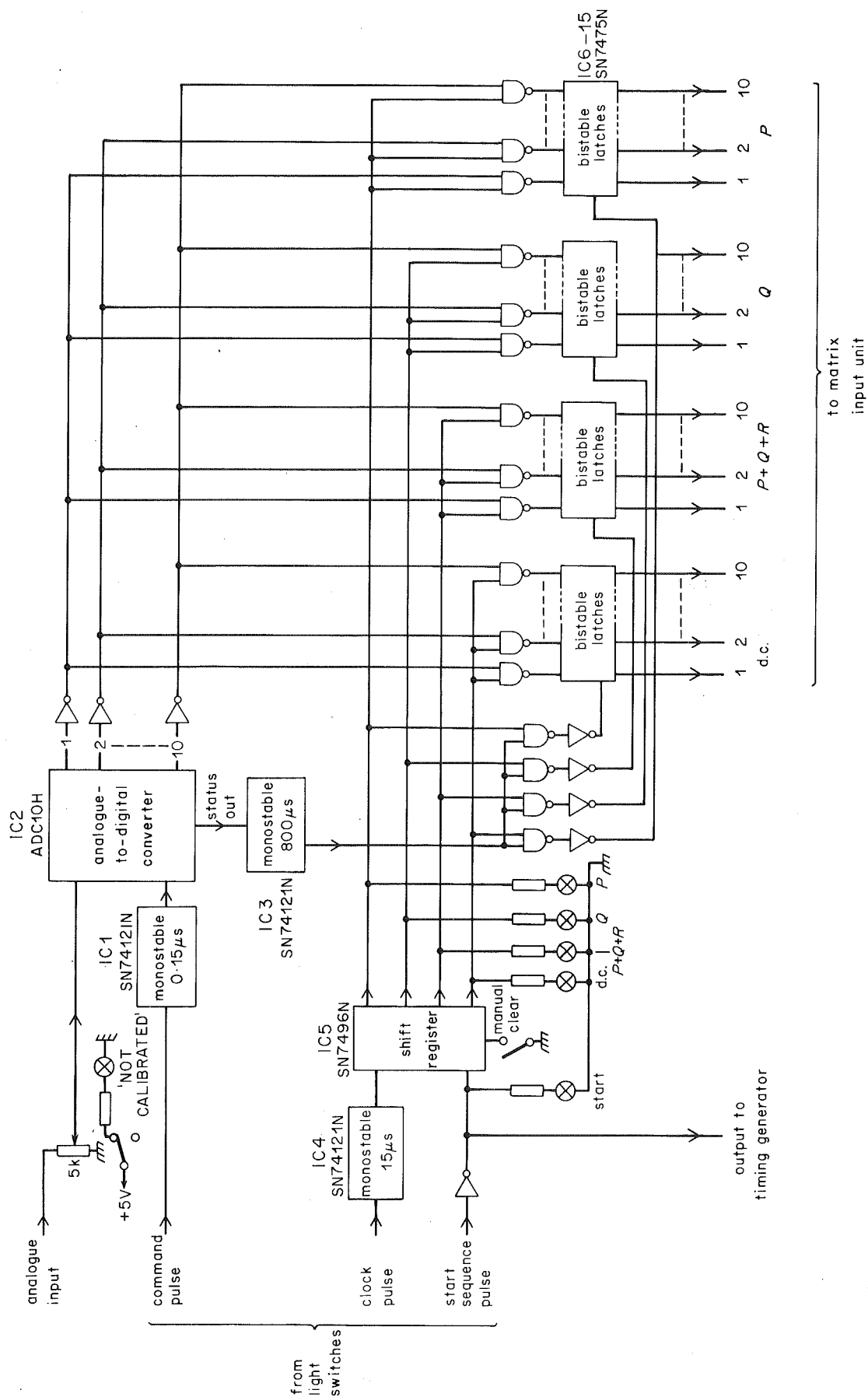


Fig. 11 - Circuit arrangement for routing and storing the binary-converted samples of the channel outputs 'dark current'  $P+Q+R$ ,  $Q$  and  $P$ .



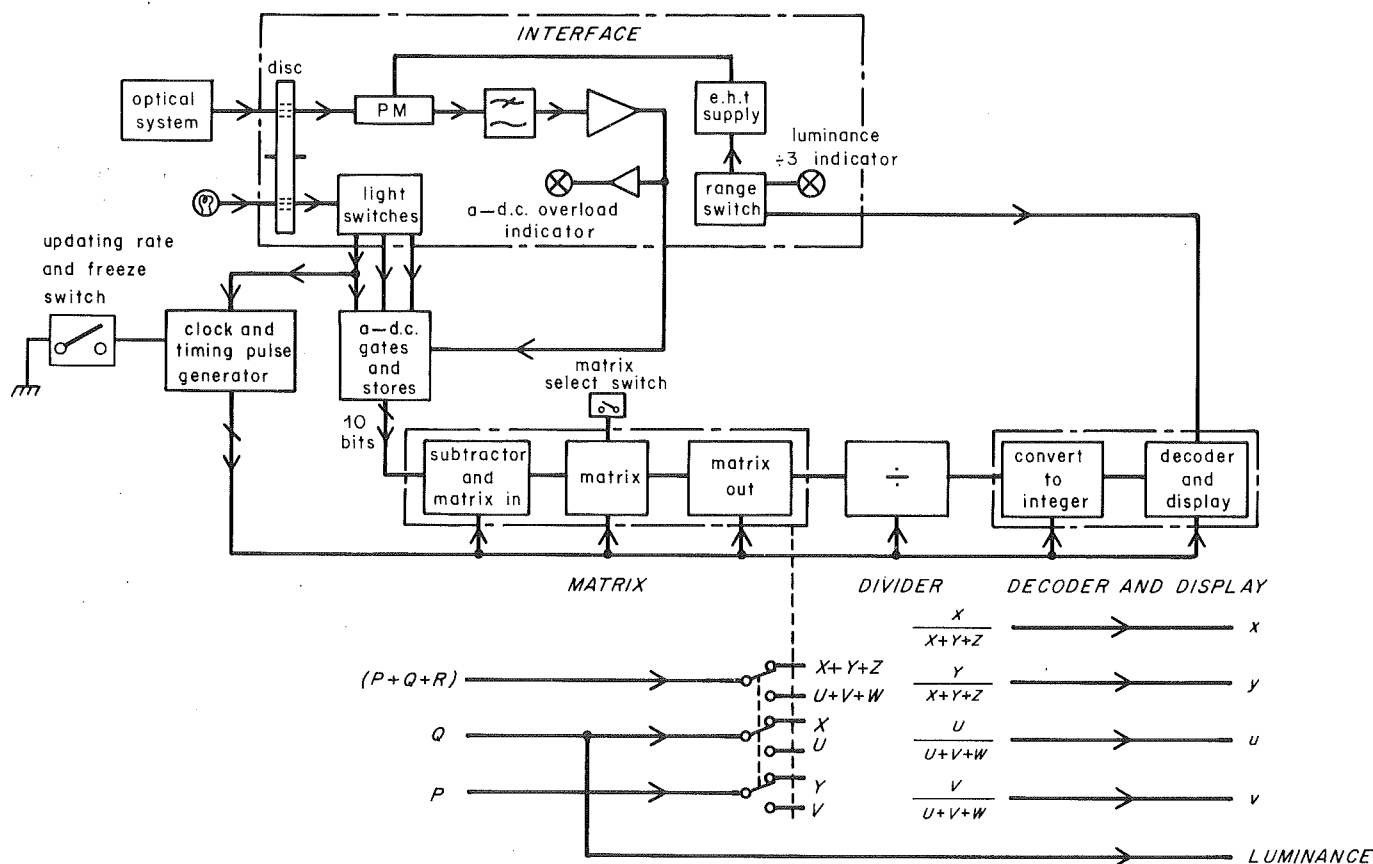


Fig. 12 - The basic functional steps carried out in the logic processor

The type of indicator used fits in a standard dual-in-line base and consists of segments of tungsten filaments; these are rated for 100,000 hours of life. A three-figure numerical display of the luminance of the viewed colour is also given. The luminance is, as explained previously, directly proportional to the incoming Q value, and the latter is available directly from the Q latch prior to the linear transformation process. It is decoded for decimal presentation in the same manner as for the chromaticity co-ordinates.

The luminance value is scaled to read in foot-Lamberts, by adjusting the overall dynode voltage of the photo-multiplier and also the amplifier gain (see Section 3.3.1.)

### 3.4.3 Operating modes

To increase the versatility of the instrument, three operating modes are available by means of a 3-position switch on the main control label. These are:

1. Continuous calculation giving rapid updating: in this mode the clock controlling the logic processor is running continuously and the read-out of the

chromaticity is therefore updated at the maximum rate, i.e. four times per revolution of the scanning disc or every 300 milliseconds.

2. One calculation after each complete revolution of the scanning disc: this is the principal mode and gives a display updating rate of once per 1.2 seconds, which is just slow enough to carry out mental averaging of the read-out if the last decimal place is uncertain.
3. Freezing the read-out: this enables the read-out at a particular time to be held in the display for as long as required. The switch simply inhibits the calculator clock after a calculation has been completed.

## 4. Performance

### 4.1 Colorimetric performance

Fig. 13 shows the spectral response of the colorimeter, without the channel shaping filters. These measurements were made using a prism spectrophotometer fitted

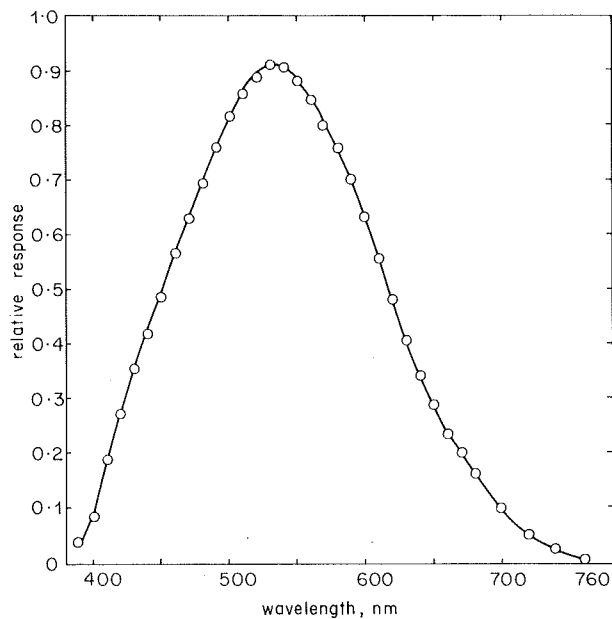


Fig. 13 - Spectral response of the colorimeter without the channel-shaping filters.

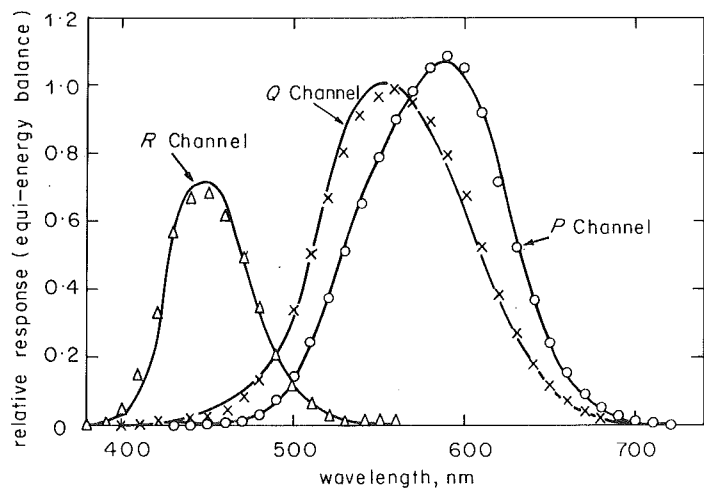


Fig. 14 - Comparison of ideal and actual channel responses for the colorimeter  
 — ideal    ○, X, △, actual (deduced from measurements)

with a thermopile detector. After measuring the relative spectral transmission curves of the shaping filter assemblies with a two-beam recording spectrophotometer, the spectral response characteristics of each channel were deduced. These were area normalised with respect to the ideal channel responses and then compared, as shown in Fig. 14, where the solid line represents the ideal case.

Considerable emphasis was placed during the optimisation of the shaping filters to ensure that the ratio of the response of the P channel to that of the Q channel was well-matched to the ideal; this is important for chromatic accuracy along the saturated green-red axis, which is particularly error-sensitive. Fig. 15 shows the degree of matching achieved; the ordinates plotted are the logarithm of the (P/Q) ratio assuming an equi-energy balance.

A more direct and positive test of the chromatic accuracy of the instrument was obtained by measuring a number of test colours of known chromaticity. A standard lamp (calibrated at the National Physical Laboratory), operating under steady direct-current at a colour temperature of  $2700^{\circ}$  K, was arranged to illuminate a neutral 'white' block (100 mm x 100 mm) uniformly. The camera head was focussed on the block at a distance of 1.5 metres. The needle attenuators in the tristimulus generator were then adjusted to give the correct chromaticity (i.e. as calculated on the basis of a Planckian radiator at  $2700^{\circ}$  K). A series of transparent colour filters was then interposed between the camera lens and the screen and

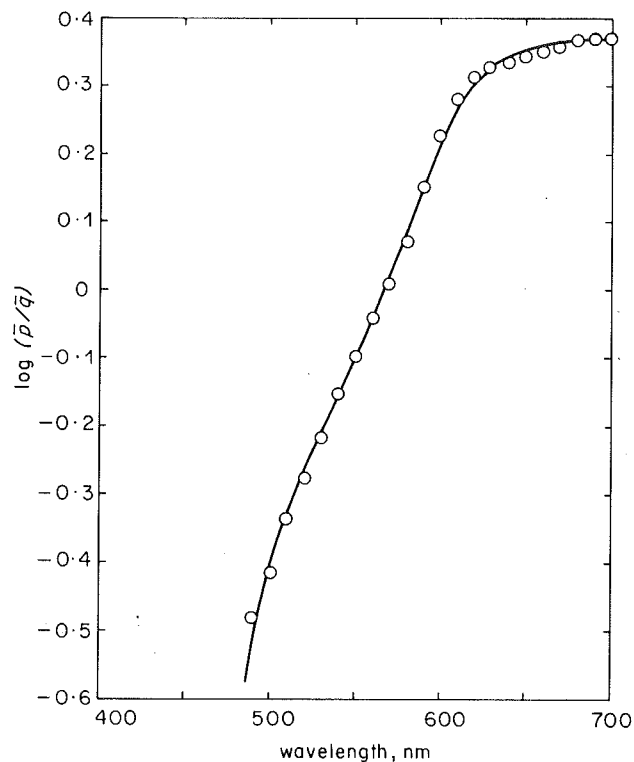


Fig. 15 - Ratio of the P channel to Q channel responses for an equi-energy balance. — ideal    ○ actual

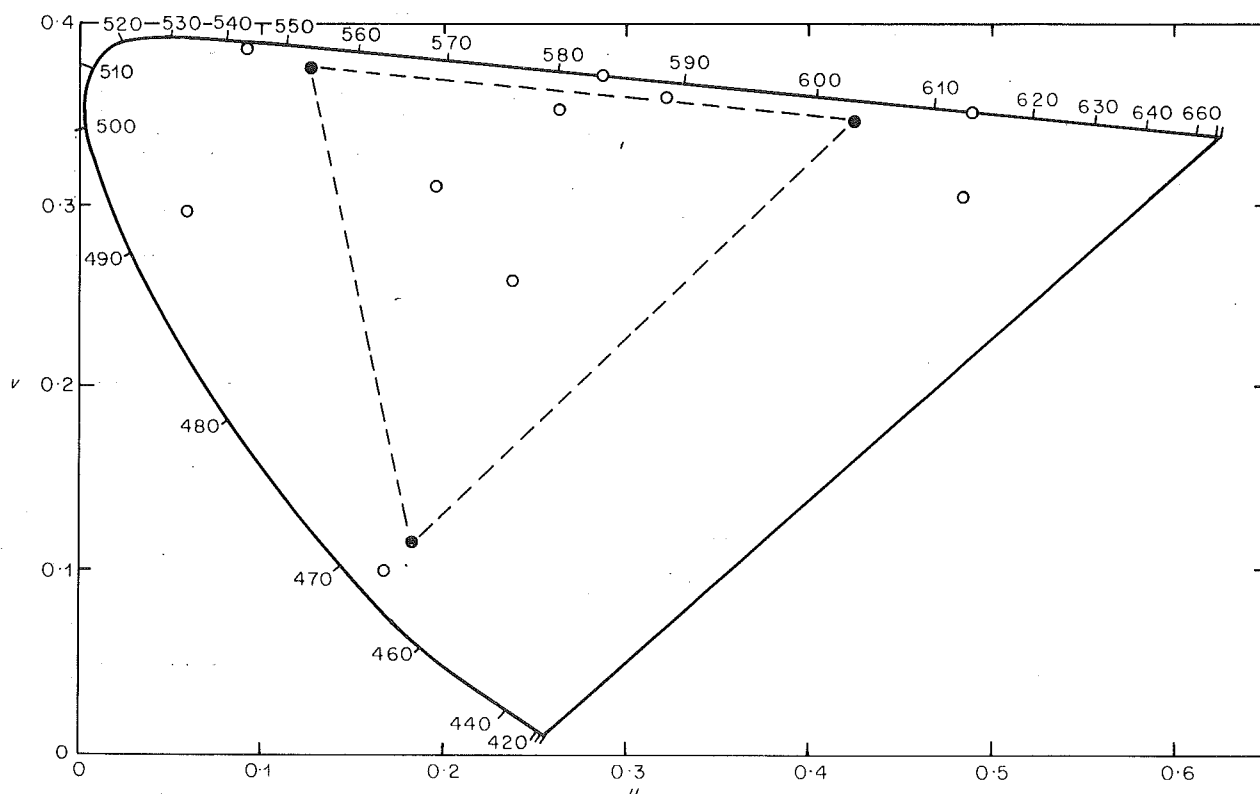


Fig. 16 - Chromaticities of test colours (nos. 1-13) on a  $(u, v)$  diagram

● Colour monitor phosphors      ○ Colour filters + std. lamp      (●-----● Gamut defined by colour television phosphors)

the chromaticity readings for each was noted. The correct chromaticities were deduced from the known spectral transmission characteristics of the test filters. The results are summarised in Table 1, which shows the calculated and measured chromaticities for a set of 13 test colours, the standard deviation of the displayed readings with respect to time, and the chromatic (vector) error of the mean values assuming the calculated values to be correct. The last three columns in the table refer to the calculated and measured luminance of the test colours the gain of the instrument being adjusted to give a displayed reading equal to the calculated luminance (40 ft. L) of the 'white' block when illuminated by the standard lamp.

The chromaticities of the 13 test colours are shown plotted on the  $u, v$  diagram in Fig. 16, which shows also the spectrum locus defining the limit of real colours. The last three colours in Table 1 were, in fact, the actual red, green and blue phosphors of a colour television monitor. These are shown in Fig. 16 as solid circles linked by a dotted line, and thus indicate the total gamut of real colours encountered in colour television reproduction. The remaining 10 test colours consisted of seven saturated or nearly saturated colours (all outside the colour television gamut), two colours on or near the locus of black-body radiators (Illuminant  $D_{65}$  and  $P_{2700}$ ) and one desaturated blue colour.

It will be seen from the above results that for all colours tested, including television monitor phosphors, the chromatic (vector) error of the instrument is less than or equal to 0.004 CIE-UCS units, i.e. less than or equal to one just noticeable difference (JND). For desaturated colours within the television gamut, the indications are that the instrument will give mean chromaticity readings which are correct to within 0.002 CIE-UCS units ( $\frac{1}{2}$  JND).

The stability of the displayed readings is limited by the shot noise associated with the primary photocathode current in the detector. As is well known, the signal-to-r.m.s. noise ratio of the signal output is proportional to the square root of the primary photocathode current. Thus the signal-to-noise ratio and hence the stability of the readings is worst for dark low-luminance colours. However, the stability of the readings is also a function of the actual chromaticity. For a given luminance, saturated reds and orange colours are the most susceptible to noise effects and blues the least (see, for example, the error curves shown in Fig. 7). In the present instrument, the average standard deviation of displayed  $(u, v)$  readings (from Table 1 and other measurements) is 0.001 ( $\frac{1}{4}$  JND) or better for colours with stable luminances greater than 40 ft-L, between 0.001 to 0.0015 for luminances in the range of 20-40 ft-L. Satisfactory results can be obtained for most colours for luminances down to 0.5 ft-L, or

Table 1:

Results for a set of test colours

Test Colour	Calculated Chromaticity				Measured Chromaticities (mean of 12 DCRM readings)				Standard Deviation of displayed chromaticity/ (UCS units)	Chromatic* Error of mean values (JND)	Absolute Luminance ft-Lamberts		
	u	v	x	y	u	v	x	y			Calcu- lated	Meas- ured	Error
1) Std. lamp 2700°K	0.2623	0.3516	0.4597	0.4108	0.2623	0.3508	0.4562	0.4079	0.0010	0.27	40	40	Gain adjusted for zero error
Approximation to 2) Illuminant D <sub>65</sub>	0.1964	0.3098	0.3078	0.3237	0.1961	0.3086	0.3063	0.3181	0.0013	0.46	4.4	4.35	0.05
3) Saturated Red	0.4899	0.3510	0.6767	0.3232	0.4918	0.3494	0.6758	0.3188	0.0020	0.84	8.73	8.6	0.13
4) Saturated Yellow	0.2861	0.3697	0.5316	0.4580	0.2875	0.3688	0.5298	0.4541	0.0014	0.36	24.5	24.5	0
5) Saturated Green	0.0923	0.3855	0.2515	0.7004	0.0941	0.3855	0.2573	0.7021	0.0017	0.49	5.1	4.95	0.15
6) Saturated Cyan	0.0598	0.2966	0.1026	0.3395	0.0585	0.3001	0.1028	0.3496	0.0021	0.9	1.97	1.9	0.07
7) Saturated Blue	0.1662	0.0985	0.1406	0.0556	0.1669	0.1018	0.1421	0.0568	0.0020	0.68	0.24	0.25	0.01
8) Sat. Magenta	0.4826	0.3041	0.5717	0.2402	0.4838	0.3034	0.5717	0.2385	0.0019	0.45	6	5.75	0.25
9) Desat. Blue	0.2391	0.2584	0.2976	0.2144	0.2394	0.2600	0.2992	0.2158	0.0016	0.36	1.8	1.72	0.08
10) Orange	0.3206	0.3587	0.543	0.405	0.3210	0.3576	0.5408	0.4014	0.0015	0.30	26	25.7	0.3
11) Red monitor phosphor	0.4249	0.3463	0.6130	0.3331	0.4260	0.3478	—	—	0.0029	0.49	—	—	—
12) Green monitor phosphor	0.1285	0.3747	0.3061	0.5953	0.1264	0.3747	—	—	0.0032	0.5	—	11	—
13) Blue monitor phosphor	0.1814	0.1140	0.2577	0.0661	0.1806	0.1101	—	—	0.0011	1.0	—	1.22	—

\*The average of u, v and x, y measurements; 1 JND = 0.004 CIE-UCS units.

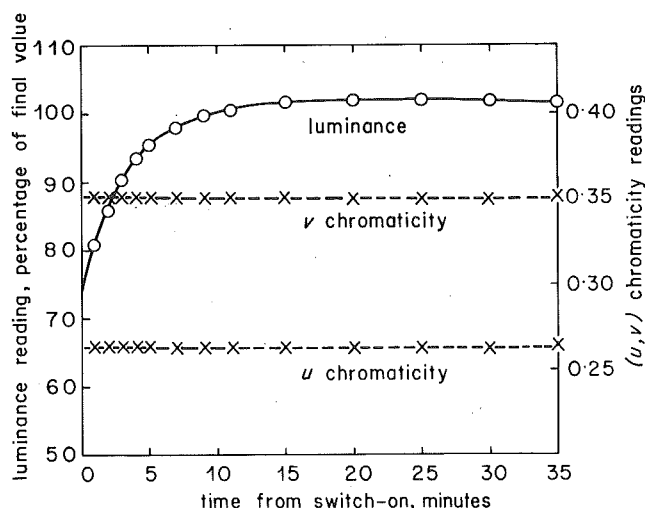


Fig. 17 - Switch-on characteristics of the colorimeter (○—○) luminance (X-----X) chromaticity

even down to 0.25 ft-L for blue colours (see results for colour No. 7 in Table 1).

The above degree of stability is only achieved if the mean luminance of the colour being viewed is sufficiently stable over a period of one or two seconds. Looking at a raster-scanned television monitor, for example, can produce a greater fluctuation of readings if the field-scan rate differs from the local mains frequency by one or two cycles and there are significant hum effects in the monitor.

Again referring to the results in Table 1, the errors in the absolute luminance readings are within 5% of the calculated values, the average error for nine test colours being approximately 2.6%. (Part of this error is due to the difference in the illumination geometry between that used in the spectrophotometer when calibrating the filters and that of the test measurement using the standard lamp.) These results, for absolute luminance, hold for the instrument after a recent gain calibration using a known standard. It is expected that after long intervals, say several months without re-adjustment, or if the instrument is used in extremes of ambient temperature, the absolute calibration accuracy may suffer due to photocathode ageing, etc.

#### 4.2 Electrical performance

The switch-on characteristic of the instrument is shown in Fig. 17, where the readings displayed when measuring a 'white' surface illuminated by a stable lamp are plotted as a function of time from switch-on. The chromaticity readings are available in a few seconds and thereafter remain constant, but the absolute luminance reading takes several minutes to stabilise. This is due to the time taken for the Zenner-diode circuits, stabilising the E.H.T. supply, to reach their operating temperature. As shown in Fig. 17, the absolute luminance reading increases to within 5% of its final value in approximately six minutes; there is then a small overshoot before the

reading settles down. The reading also drifts slightly when a new range is selected but soon re-stabilises.

Fig. 18 shows the response/frequency characteristic of the low-pass filter network followed by the differential amplifier. It will be seen that the attenuation of 50 Hz components is approximately -80 dB, and the 25 Hz components -46 dB, with respect to the low frequencies commensurate with the principal sampling rate.

The first integrated-circuit differential amplifier is designed for instrumentation work and features an extremely low drift of the input offset voltage. The offset balance has been pre-set so that the total offset voltage at the input to a.d. convertor is in the range 20-40 millivolts,\* and this should not require re-setting over long periods. On the most sensitive range of the photo-multiplier, the residual dark current is less than 40 nano-amperes at normal ambient temperatures. This maximum dark current gives rise to an additional voltage at the input to the a.-d. convertor of approximately 100 millivolts. On the least sensitive ranges the additional voltage is of the order of 1 millivolt.

#### 5. General assessment

A general specification of the characteristics of the colorimeter is given in Appendix III of this report.

The principal aim, to develop a continuously reading colorimeter of high chromatic accuracy, has been achieved. It is expected that the instrument will be a potent aid to analytical investigations concerned with colour television broadcasting. While maintaining the same general principles, there are some aspects of the present design which could benefit from further development. In particular, the

\*The output offset voltage must not become negative otherwise the dark-current correction circuits in the calculator are not then operative and an error will result.

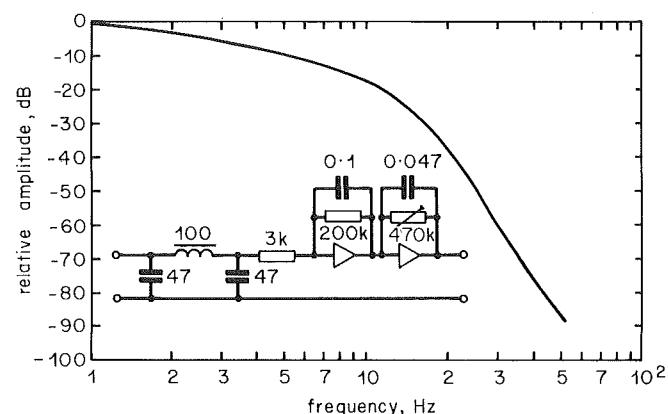


Fig. 18 - Response/frequency characteristic of the low-pass filter and differential amplifier used in the interface circuit.

light transmission losses that occur with the present fibre-optic arrangement could be significantly reduced, for example by using a better quality fibre and by reducing the number of couplings. Moreover, a spatially randomised fibre-optic lead would result in a better integration of the light from the area being measured and minimise the effect of luminance shading.

The computer optimisation of the shaping filter glasses has been a great help towards achieving spectral responses close to the ideal. However, to obtain the best results it was found necessary finally to make slight empirical corrections to the filter responses, based on the actual readings obtained for a range of test colours of known chromaticity. In the red part of the chromatic diagram, the fidelity of the channel responses required to maintain an accuracy better than 1 JND is very high indeed. For example, replacing one of the optical lenses in the instrument by another having a slightly different transmission characteristic, due to a different anti-reflection coating say, is sufficient to throw the instrument out of calibration. Also, any ageing of the spectral response of the photo-

multiplier detector will have a similar deleterious effect and careful selection of this component is therefore imperative if the long-term accuracy is to be maintained.

As pointed out earlier, no emphasis was placed on such aspects as minimising the bulk and weight of the instrument. Indeed, since the start of the project, more sophisticated solid-state components have become commercially available which could dramatically reduce the size of the hardware necessary to carry out the digital computation. Thus there would be considerable scope for stream-lining in any future version if desired.

A photograph of the direct-reading colorimeter is shown in Fig. 19.

## 6. References

1. PHILIPPART, H.A.S. 1966. A tri-stimulus spot colorimeter. BBC Engineering Monograph, No. 65, December, 1966.



*Fig. 19 - The complete instrument as used in a studio environment*

2. WRIGHT, W.D. 1964. The measurement of colour. London, Hilger and Watts, 3rd edition, 1964.
3. WRIGHT, H., SANDERS, C.L. and GIGNAC, G. 1969. Design of glass filter combinations for photometers. *Applied Optics*, 8, 12, December, 1969, pp. 2449 to 2455.

### Appendix I

#### Equations connecting the X, Y, Z; U, V, W and P, Q, R sets of Tristimulus values

Table 2 shows the chromaticity co-ordinates of the sets of reference primaries (X) (Y) (Z), (U) (V) (W) and (P) (Q) (R) to which the sets of tristimulus values refer, respectively.

Table 2  
Chromaticities of reference primaries

	(X)	(Y)	(Z)	(U)	(V)	(W)	(P)	(Q)	(R)
<i>u</i>	4	0	0	1	0	0	4	$-\frac{4}{19}$	$\frac{1}{4}$
<i>v</i>	0	$\frac{2}{5}$	0	0	1	0	0	$\frac{8}{19}$	0
<i>w</i>	-3	$\frac{3}{5}$	1	0	0	1	-3	$\frac{15}{19}$	$\frac{3}{4}$

The basic matrix equations relating these sets of tristimulus values are:

$$\begin{bmatrix} U \\ V \\ W \end{bmatrix} = \begin{bmatrix} \frac{2}{3} & 0 & 0 \\ 0 & 1 & 0 \\ -\frac{1}{2} & \frac{3}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \quad \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} \frac{3}{2} & 0 & 0 \\ 0 & 1 & 0 \\ \frac{3}{2} & -3 & 2 \end{bmatrix} \begin{bmatrix} U \\ V \\ W \end{bmatrix}$$

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} \frac{3}{2} & -\frac{3}{4} & \frac{1}{2} \\ 0 & 1 & 0 \\ 0 & 0 & \frac{5}{2} \end{bmatrix} \begin{bmatrix} P \\ Q \\ R \end{bmatrix} \quad \begin{bmatrix} P \\ Q \\ R \end{bmatrix} = \begin{bmatrix} \frac{2}{3} & \frac{1}{2} & -\frac{2}{15} \\ 0 & 1 & 0 \\ 0 & 0 & \frac{2}{5} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

$$\begin{bmatrix} U \\ V \\ W \end{bmatrix} = \begin{bmatrix} 1 & -\frac{1}{2} & \frac{1}{3} \\ 0 & 1 & 0 \\ -\frac{3}{4} & \frac{15}{8} & 1 \end{bmatrix} \begin{bmatrix} P \\ Q \\ R \end{bmatrix} \quad \begin{bmatrix} P \\ Q \\ R \end{bmatrix} = \begin{bmatrix} \frac{4}{5} & \frac{9}{10} & -\frac{4}{5} \\ 0 & 1 & 0 \\ \frac{3}{5} & -\frac{6}{5} & \frac{4}{5} \end{bmatrix} \begin{bmatrix} U \\ V \\ W \end{bmatrix}$$

Additionally, we have the modified versions, as used in the colorimeter.

$$\begin{bmatrix} U \\ V \\ U+V+W \end{bmatrix} = \begin{bmatrix} \frac{2}{3} & -\frac{5}{6} & \frac{1}{3} \\ 0 & 1 & 0 \\ -\frac{13}{12} & \frac{25}{24} & \frac{4}{3} \end{bmatrix} \begin{bmatrix} P \\ Q \\ P+Q+R \end{bmatrix} \quad \begin{bmatrix} X \\ Y \\ X+Y+Z \end{bmatrix} = \begin{bmatrix} 1 & -\frac{5}{4} & \frac{1}{2} \\ 0 & 1 & 0 \\ -\frac{3}{2} & -\frac{4}{11} & 3 \end{bmatrix} \begin{bmatrix} P \\ Q \\ P+Q+R \end{bmatrix}$$

## Appendix II

### Error functions for P, Q, R set of Tristimulus values

Suppose that we have a set of tristimulus values referring to the amounts of a specific set of reference primaries which are additively combined to form a mixture colour, real or otherwise. This colour has a chromaticity  $(U_m, V_m)$  say, but if now one of the tristimulus values is perturbed by a small fractional amount  $\pm a$ , the new mixture chromaticity will be slightly different  $(U_m + \delta U, V_m + \delta V)$ . The chromatic (vector) displacement from the original chromaticity is, by definition  $\delta(u, v) = [(\delta u)^2 + (\delta v)^2]^{1/2}$ , and this displacement is a function of the original mixture chromaticity  $(U_m, V_m)$ .

It may be shown that, for the (P), (Q) and (R) reference primaries used in the colorimeter, the displacement  $\delta(u, v)$  which occurs when one (the one indicated by the subscript) of the tristimulus quantities is perturbed is given by:

$$\delta_P(u, v) = \frac{a}{120} (32 u_m + 35 v_m - 8) [(u_m - 4)^2 + v_m^2]^{1/2}$$

$$\delta_Q(u, v) = \frac{a}{8} v_m [(19 u_m + 4)^2 + (19 v_m - 8)^2]^{1/2}$$

$$\delta_R(u, v) = \frac{a}{15} (4 - u_m - 10 v_m) [(4 u_m - 1)^2 + (4 v_m)^2]^{1/2}$$

These equations can be used to determine the relative sensitivity of any colour to chromatic errors caused by perturbations or errors in the individual tristimulus amounts. In the main text, for example, Fig. 7 shows the calculated error distribution for narrow-band spectrum colours, assuming a 2% error in one of the tristimulus amounts.



### Appendix III

#### General specification of experimental colorimeter

1.	Field of view	2° with $f = 135$ mm lens. (Lenses interchangeable if transmission characteristics are matched.)			
2.	Display Modes (switch selected)	1) Chromaticity readings updated after every channel sample, i.e. every 0.3 seconds 2) Chromaticity and luminance readings updated after every complete sampling sequence, i.e. every 1.2 seconds 3) All readings 'frozen'.			
3.	Read-out				
	a) <u>Luminance</u> Reading Effective range Absolute accuracy	3 digits with decimal point coupled to gain switch and 'divide-reading-by-three' instruction on intermediate range positions. 0.05 to 999 ft. Lamberts in six ranges. $\pm 5\%$ at 1.5 m from surface			
	b) <u>Chromaticity</u> Readings Lowest-luminance Chromatic accuracy (vector error)	3 decimal places in either CIE ( $u, v$ ) or CIE ( $x, y$ ) co-ordinates (switch selected). 0.2 to 0.5 ft. Lamberts depending on colour. $< 0.005$ CIE-UCS units. ( $0.004$ CIE-UCS units over colour television gamut).			
	c) <u>Stability and Initial Drift</u> ( $u, v$ ) chromaticity reading standard deviation (CIE-UCS units)	0.001, $> 40$ ft. L. 0.0015, 20–40 ft. L.                      or better depending 0.002, 5–20 ft. L.                          on colour 0.003, $\frac{1}{2}$ –5 ft. L.			
	Luminance Reading	Readings available within 30 seconds from power supply switch-on $\pm 5\%$ of final value reached within 7 minutes from power supply switch-on.			
4.	Power Consumption	138 W			
5.	Size	height	width	depth	weight
		mm	mm	mm	kg
	Camera head and tripod	600-1800	—	—	7.8
	Tristimulus generator	175	180	440	7.8
	Calculator and display unit	285	400	500	17
	Fibre-optic cable length	1800 mm			
6.	ADC	10-bit resolution, 5V input			

